Elastic wave propagation mechanisms in underwater acoustic environments

Scott D. Frank
Marist College
Department of Mathematics
Poughkeepsie, NY 12601
phone: (845) 575-3977 fax: (845) 575-3605 email: scott.frank@marist.edu

Award Number: N00014-14-1-0640

LONG-TERM GOALS

Develop elastic parabolic equation (EPE) method capabilities in order to characterize effects of elastic propagation mechanisms such as elastic interface scattering, conversion from elastic propagation to acoustic propagation, and intense interface waves on underwater acoustic environments with elastic bottoms or elastic ice cover.

OBJECTIVES

To apply EPE solutions to scenarios that include fluid-elastic boundaries, either at the ocean floor, or at the ocean surface as an elastic ice layer. Computational tools will be developed or enhanced to characterize the transmission of elastic wave energy to acoustic energy in the water column. Elastic material parameters will be varied for analysis of the dissipation of water column acoustic energy resulting from interaction with elastic layers. In particular, oceanic T-waves, Scholte interface waves, and plate flexural waves of an elastic ice layer are geophysical mechanisms whose effect on acoustic transmission loss will be evaluated. The impact of range-dependent seafloor or ice layers on acoustic propagation will be considered as a means to predict the presence of elastic ice layers.

APPROACH

In a cylindrically symmetric environment, where r is the distance from the source and z is depth, recent parabolic equation methods for acoustic propagation in elastic sediments are based on the (u_r, w) formulation of elasticity, where u_r is the horizontal derivative of the horizontal displacement and w is the vertical displacement.[1] The outgoing portion of the separated Helmholtz operator leads to the parabolic equation for a range-independent environment,

$$\frac{\partial}{\partial r}\begin{pmatrix} u_r \\ w \end{pmatrix} = i(L^{-1}M)^{1/2}\begin{pmatrix} u_r \\ w \end{pmatrix}, \quad u_r = \frac{\partial u}{\partial r}, \tag{1}$$

where L and M are matrices containing depth-dependent operators that incorporate compressional wave speed, c_p, shear wave speed, c_s, and density ρ via the Lamé parameters of the elastic medium, λ and μ.

Range dependence is included by modeling sloping boundaries with a stair-step approximation and applying appropriate matching conditions at each vertical interface.

Elastic parabolic equation solutions have been generated for a single fluid-elastic interface occurring at the ocean bottom. To incorporate the effects of an elastic ice layer into these solutions, additional boundary conditions must be explicitly enforced during calculation of the acoustic field. In particular, the ice-air interface requires ta zero traction condition, expressed as vanishing normal and tangential stresses. These conditions can be written

$$\lambda(u_r) + (\lambda + 2\mu)\frac{\partial w}{\partial z} = 0, \tag{2}$$

and

$$\frac{\partial}{\partial z}(\lambda(u_r)) + \frac{\partial}{\partial z} \quad (\lambda + 2\mu)\frac{\partial w}{\partial z} \quad + \rho\omega^2 w = 0. \tag{3}$$

which are then discretized using a Galerkin method.[1] The ice-water interface requires the same conditions as the water-seafloor interface: continuity of vertical displacement, continuity of normal stress, and continuity of tangential stress.

Wavenumber content of an EPE solution can be used to resolve modes present in the solution and determine if they are propagating acoustic modes, leaky acoustic modes, interface wave modes, or represent other types of elastic propagation. The wavenumber spectra are calculated using the Hankel transform of the range-dependent acoustic field solution.

Elastic normal mode solutions were obtained using a Green's function approach that has been shown to be accurate for underwater acoustic environments with elastic ocean bottoms.[2] The addition of a range independent ice layer leads to a complicated characteristic equation for horizontal wavenumbers that is solved using a winding integral technique.[3] Modal solutions obtained using these wavenumbers compare accurately to wavenumber integration solutions for range-independent environments and can be used for analysis of mode shapes and determination of elastic mode behavior, for example as a leaky mode or interface mode.

WORK COMPLETED

- Elastic PE solutions with deep seismic sources[4] demonstrate generation and propagation of oceanic T-waves and Scholte interface waves at the ocean bottom.[5]
- The capability of EPE solutions to generate accurate solutions in an ice-covered environment for range-independent environments was established.[3]
- Horizontal wavenumber spectra obtained from EPE solutions reveal an excited flexural mode that propagates in the ice layer at certain acoustic frequencies in ice-covered environments.[3]
- Previously implemented EPE self-starters[4] generate solutions in complicated range and depth dependent beach and island propagation scenarios and demonstrate conversion of energy from purely elastic propagation to acoustic propagation and back.[6]

RESULTS

Elastic parabolic equation results were benchmarked against elastic normal mode solutions in a range-independent elastic ice-covered environment with an elastic ocean bottom. Horizontal wavenumbers for a 20 Hz source in the water layer are shown in Fig. 1(a) for an example with a 10 m ice layer as open circles. Dashed lines indicate wavenumbers $k_1, k_2, k_{s,2}, k_3$, and $k_{s,3}$, corresponding to the water layer sound speed, bottom compressional and shear wave speed, and ice layer compressional and shear wave speed respectively. Propagating acoustic modes occur between k_1 and k_2. The wavenumber furthest to the right corresponds to the plate flexural mode of the ice layer.[7] This wavenumber has a calculated wave speed of 795.0 m/s, which is consistent with a theoretically predicted wave speed of 782.7 m/s for a plate flexural mode with the parameters in this example.[8] The other wavenumber to the right of $k_{s,2}$ has a wave speed approximately 82% of the bottom shear speed and corresponds to a Scholte wave. Neither of these modes would occur in a fluid-based solution.

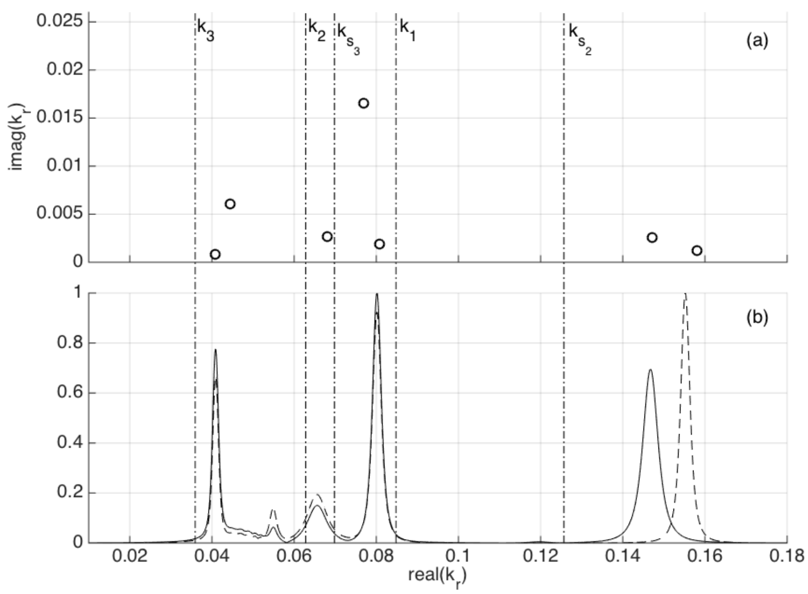

Figure 1: (a) Horizontal wavenumbers (open circles) from elastic normal mode solution for a 20 hz source in a range independent environment with a 10 m thick ice layer and 100 m thick isospeed water layer over an elastic bottom. Dashed lines correspond to relevant compressional and shear wave speeds in the three media. The wavenumber furthest to the right corresponds to a plate flexural mode of the ice layer and has a wavespeed consistent with physical theory. The other wavenumber to the right of k_2 corresponds to a Scholte interface mode propagating at the ocean bottom.(b) Solid curve shows horizontal wavnumber spectra obtained from parabolic equation solution for a source at 99 m under the ice layer and reciever at 85 m. This source receiver configuration shows the excitation of the Scholte interface mode. Dashed curve shows spectra for a source at 1 m depth and receiver at 25 m, showing the excitation of the plate flexural mode.

Figure 2 shows elastic parabolic equation solution results for a 75 Hz source placed at $r = 0$ m and source depth 55 m in an environment with range-dependent ice and bottom interfaces. The ice-water interface varies from 1 to 2 m thickness over the 12 km range shown. The water column has a range independent double-duct sound speed profile shown in Fig. 2(a), which is similar to one obtained from a

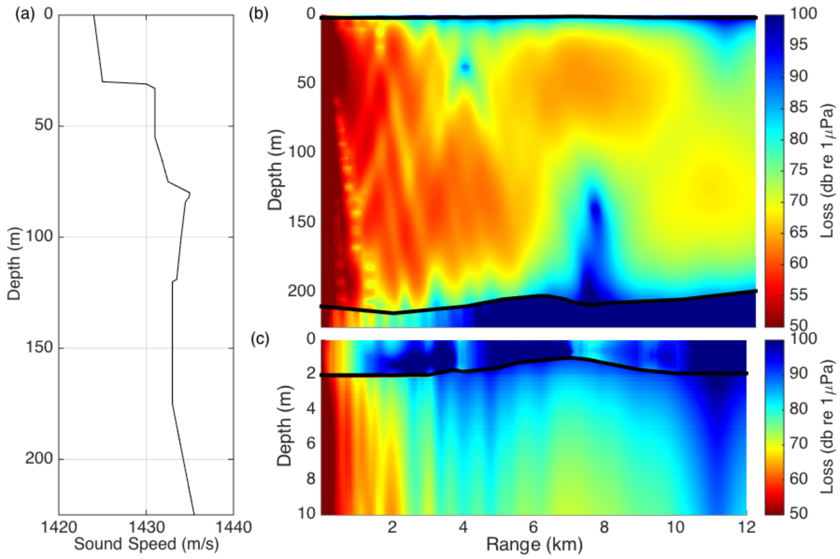

Figure 2: (a) A double-ducted Arctic profile similar to an experimental CTD cast from the Beaufort Sea. (b) Elastic parabolic equation solution using the profile in (a) for an environment with an elastic bottom layer and elastic ice cover. A 75 Hz acoustic source is located at $r = 0$ km and $z_s = 55$ m. Effects of the sound speed profile are evident from the concentration of acoustic energy propagating above 75 m depth. (c) Detail of the top 10 m of the contour plot in (b) illustrating acoustic propagation and conversion to elastic energy in the ice layer. At points where the ice layer thickens acoustic energy is transmitted from the water column into the elastic layer.

CTD cast in the Beaufort Sea in March 2009.[9] The ocean bottom interface is also range dependent with a depth of approximately 210 m. Both range dependent interfaces were constructed for demonstration. Transmission loss contour plot is shown in Fig.2(b). The effect of the double-duct is evident at approximately 3 km range, where acoustic energy appears trapped above 50 m. Figure 2(c) shows a detail of ice layer and shallower depths of (b). Acoustic energy is scattered from the water into the ice layer when it is increasing in thickness, specifically near 4 and 7 km.

IMPACT/APPLICATIONS

- Improved modeling capabilities of elastic parabolic equation methods for underwater acoustic problems where elastic properties of the bottom or an overlying ice layer cannot be ignored. Specific cases include the generation and propagation of oceanic T-waves by seismic sources which are relevant for geophysical study or test ban treaty monitoring.
- Oceanic T-waves and interface waves are potential explanations for "deep seafloor arrivals" and the reception of acoustic signals in what may be otherwise considered a quiet ocean environment for monitoring.
- Advances in modeling acoustic propagation in elastic layers has potential application in ice covered environments where an elastic layer lies on top of the water column. These advances are relevant to inverse problems where there is a desire to remotely determine the presence or absence of an ice layer.

RELATED PROJECTS

This research relates to the separately funded work of Robert Odom (Applied Physics Laboratory, University of Washington) regarding the two-way coupled mode code and acoustic propagation in ice-covered environments. It also relates to parabolic equation development by Jon M. Collis (Colorado School of Mines).

REFERENCES

[1] Wayne Jerzak, William L. Siegmann, and Michael D. Collins. Modeling Rayleigh and Stoneley waves and other interface and boundary effects with the parabolic equation. *J. Acoust. Soc. Am.*, 117(6):3497–3503, June 2005. doi: 10.1121/1.1893245.

[2] Brittany A. McCollom and Jon M. Collis. Root finding in the complex plane for seismo-acoustic propagation scenarios with Green's function solutions. *J. Acoust. Soc. Am.*, 136(3):1036–1045, September 2014. doi:10.1121/1.4892789.

[3] Jon M. Collis, Scott D. Frank, Adam M. Metzler, and Kimberly S. Preston. Elastic parabolic equation and normal mode solutions for seismo-acoustic underwater environments with ice covers. *J. Acoust. Soc. Am.*, Submitted May 2015.

[4] Scott D. Frank, Robert I. Odom, and Jon M. Collis. Elastic parabolic equation solutions for elastic bottom underwater acoustic problems using seismic sources. *J. Acoust. Soc. Am.*, 133(3):1358–1367, 2013.

[5] Scott D. Frank, Jon M. Collis, and Robert I. Odom. Elastic parabolic equation solutions for oceanic T-wave generation and propagation from deep seismic sources. *J. Acoust. Soc. Am.*, 137(6):3534–3543, June 2015.

[6] Scott D. Frank and Jon M. Collis. Seismic sources in parabolic equation solutions for beach and island propagation scenarios (a). *J. Acoust. Soc. Am.*, 137(4):2243, 2015.

[7] Kevin LePage and Henrik Schmidt. Modeling of low-frequency transmission loss in the central Arctic. *J. Acoust. Soc. Am.*, 96(3):17831795, September 1994.

[8] Erland M. Schulson. The structure and mechanical behavior of ice. *JOM*, 51(2):21–27, February 1999.

[9] Lee Freitag, Peter Koski, Andrey Morozov, Sandipa Singh, and James Partan. Acoustic communications and navigation under Arctic ice. In *Oceans, 2012*, pages 1–8. IEEE, October 2012. 10.1109/OCEANS.2012.6405005.

PUBLICATIONS

- Published in refereed journal *(JASA, 2015)*: [5]
- Submitted to refereed journal (*JASA*): [3]
- Presentations: [6]